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The Life Duration for Compressor Disc under Torsional Vibrations of the Blades

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Abstract. The purpose of the research is calculating the stress-strain state of the elastic compressor disks of gas turbine engine due to vibrations of blades. The thickness of the disks is variable along the radius. The solution presented by power and Fourier series and the Fourier coefficients are found from the boundary value problems for systems of ordinary differential equations along the radial coordinate. The obtained results are used to estimate the durability of the disks in very-high-cycle fatigue mode.

Introduction

The compressor disks of a gas turbine are loaded by the oscillating blades and this may lead to failure in the very-high-cycle fatigue mode (VHCF) [1]. The frequency of these oscillations is of the order of disc rotation frequency, or a multiple of it. Development of VHCF with the number of cycles to failure $N>10^8$ may cause appearance of failure in the vicinity of the contact area of the blades and the outer rim of the disc. Earlier in [2] for low-cycle fatigue regime (LCF) the stress-strain state and the fatigue durability estimation were calculated for a rotating disk of variable thickness under the action of centrifugal loads in the disk and blades and aerodynamic loads of air flow on blades. Such loads correspond to flight cycles (takeoff-flight-landing). However, along with the LCF mode (flight cycles) there are also acting the low-amplitude cyclic loads of VHCF mode due to blade vibrations. A distinctive feature of VHCF mode is that the center of damage zone is situated under the surface of the structural element. These features allow experimenters to distinguish between these mechanisms in the classification of the primary damage reason. A review of experimental research in this field can be found in [1, 3].

The purpose of the given work consists in an estimation of life duration of a disk of variable thickness in VHCF mode (vibrations) together with LCF mode. For this the stresses due to vibrations of blades are imposed on the stress field due to flight cycles. It is assumed that the vibrations are known beforehand according to observation data on amplitudes and frequencies during exploitation presented in [1]. Full stress state from flight cycles and vibrations for two extreme positions of blade at vibrating torsion are the borders of studied cyclic process and these values are used in criteria of fatigue durability. On the basis of multiaxial fatigue criterion the time to fracture and the zones of fatigue failure origin are found. There are no experimentally proved, standard criteria for time to failure detection in VHCF mode. Therefore for estimations of VHCF fracture durability ($N>10^8$), Fig. 1a, the generalization [4] of known multiaxial LCF failure criterion [2] is used.

1. The approximate system of the equations for a disk of variable thickness under action of periodic loadings on external disk boundary. Let in cylindrical system of coordinates r , ϑ , z the ring disk $a \le r \le b$ has variable thickness $2h(r)$. The coordinate along thickness varies in limits $-h(r) \le z \le h(r)$. The equations of dynamic theory of elasticity in cylindrical coordinates have the following view:

$$\frac{\partial \sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_{r\vartheta}}{\partial \vartheta}+\frac{\partial \sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\vartheta\vartheta}}{r}=\rho\frac{\partial^2 u_r}{\partial t^2} \qquad \frac{\partial \sigma_{r\vartheta}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_{\vartheta\vartheta}}{\partial \vartheta}+\frac{\partial \sigma_{\vartheta z}}{\partial z}+\frac{2\sigma_{r\vartheta}}{r}=\rho\frac{\partial^2 u_\vartheta}{\partial t^2}$$

$$\frac{\partial \sigma_{rz}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_{\vartheta z}}{\partial \vartheta}+\frac{\partial \sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}=\rho\frac{\partial^2 u_z}{\partial t^2}$$

Stresses and strains are subjected to Hooke's law. Relations between strains and displacements are:

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r} \qquad \varepsilon_{\vartheta\vartheta}=\frac{1}{r}\frac{\partial u_\vartheta}{\partial \vartheta}+\frac{u_r}{r} \qquad \varepsilon_{r\vartheta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \vartheta}+\frac{\partial u_\vartheta}{\partial r}-\frac{u_\vartheta}{r}\right) \qquad \varepsilon_{\vartheta z}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_z}{\partial \vartheta}+\frac{\partial u_\vartheta}{\partial z}\right)$$

where λ, μ are Lame elastic moduli, ρ is density of disk material. The boundary conditions on free boundaries at $z=\pm h(r)$ are:

$$\sigma_{rz}-h'\sigma_{rr}=0 \qquad \sigma_{\vartheta z}-h'\sigma_{r\vartheta}=0 \qquad \sigma_{zz}-h'\sigma_{rz}=0$$

Internal boundary of disk is unloaded:

$$r=a: \quad \sigma_{rr}=0 \quad \sigma_{r\theta}=0 \quad \sigma_{rz}=0$$

On the external disk boundary the periodic (along the angular coordinate ϑ and time t) applied loads $\sigma_{r\theta\_b}$ and σ_{rz_b} are produced by torsional vibrations of the blades:

$$r=b: \quad \sigma_{rr}=0 \quad \sigma_{r\theta}=\sigma_{r\theta\_b} \quad \sigma_{rz}=\sigma_{rz\_b}$$

Because of the periodicity of unknown functions along the angular coordinate ϑ and time t the displacements and stresses are represented in the form of Fourier series:

$$u_r=e^{i\omega t}\sum_{n=1}^{\infty}(u_n z+u_{3n}z^3)\sin n\vartheta \quad u_\vartheta=e^{i\omega t}\sum_{n=0}^{\infty}(v_n z+v_{3n}z^3)\cos n\vartheta \quad u_z=e^{i\omega t}\sum_{n=1}^{\infty}(w_n+w_{2n}z^2+w_{4n}z^4)\sin n\vartheta$$

$$\sigma_{rr}=e^{i\omega t}\sum_{n=1}^{\infty}(\sigma_n z+\sigma_{3n}z^3)\sin n\vartheta \qquad \sigma_{\vartheta\vartheta}=e^{i\omega t}\sum_{n=1}^{\infty}(s_n z+s_{3n}z^3)\sin n\vartheta \qquad \sigma_{zz}=e^{i\omega t}\sum_{n=1}^{\infty}(\Sigma_n z+\Sigma_{3n}z^3)\sin n\vartheta$$

$$\sigma_{r\vartheta}=e^{i\omega t}\sum_{n=0}^{\infty}(\tau_n z+\tau_{3n}z^3)\cos n\vartheta \qquad \sigma_{rz}=e^{i\omega t}\sum_{n=1}^{\infty}(p_n+p_{2n}z^2+p_{4n}z^4)\sin n\vartheta \qquad \sigma_{\vartheta z}=e^{i\omega t}\sum_{n=0}^{\infty}(T_n+T_{2n}z^2+T_{4n}z^4)\cos n\vartheta$$

The coefficients of the Fourier series are new unknown functions of the radial variable r, multiplier $e^{i\omega t}$ defines vibrations, ω is a vibration frequency. If substitute the expressions for displacements and stresses in the original system and equate terms of like powers z up to z^3 then one can obtain the system of ordinary differential equations for the auxiliary variables at different n.

2. The boundary conditions for torsional vibrations. For calculations of stress state for disk due to torsional vibrations of blades the boundary conditions for auxiliary variables (Fourier coefficients) on radial boundaries $r=a$ and $r=b$ are

$$r=a: \quad \sigma_n=0,\ \sigma_{3n}=0,\ \tau_n=0,\ \tau_{3n}=0,\ p_n=0,\ p_{2n}=0$$

$$r=b: \quad \sigma_n=0,\ \sigma_{3n}=0,\ \tau_n=\tau_{bn},\ \tau_{3n}=0,\ p_n=p_{bn},\ p_{2n}=-p_{bn}/h^2$$

where τ_{bn} and p_{bn} are predefined values of Fourier coefficients for disk boundary stresses in root sections of blades undergoing to torsion. For calculation of values τ_{bn} and p_{bn} consider every blade as a plate of rectangular section of width d, torsion of intensity γ and use the approximate solution of known task about torsion of plates of rectangular cross section:

$$\tau_b(\vartheta)=Q_0\left(1-\vartheta^2/\delta^2\right),\ Q_0=-0.8\mu\gamma d/h, \quad p_b(\vartheta)=T_0\,\vartheta/\delta, \quad T_0=\mu\gamma d, \quad |\vartheta|\le\delta, \quad \delta=d/(2b)<<1$$

Let the number of blades on the disk is equal to N_0. Expand the periodic distribution function of the tangential stress $\sigma_{r\vartheta\_b}$ and axial shear stress σ_{rz_b} on external boundary (for $r=b$) in Fourier series (one period $-\pi/N_0<\vartheta<\pi/N_0$), $n=N_0, 2N_0, \ldots$

$$\tau_b(\vartheta)=\sum_{k=0}^{\infty}\tau^{(k)}\cos\left(kN_0\vartheta\right) \qquad p_b(\vartheta)=\sum_{k=1}^{\infty}p^{(k)}\sin\left(kN_0\vartheta\right)$$

Thus, for various n there must be solved two-point boundary value problems for systems of ordinary differential equations with boundary conditions. The solution of these boundary value problems are determined by Godunov's orthogonal sweep method.

3. Examples of calculation of disk stress state due to vibrations. The half-section of disk at $\vartheta = const$ is shown in Fig. 1-b. Accepted the following values of geometry and material parameters for disk and blades: $a = 0.05$m, $b = 0.4$m, $d = 0.01$m, h=0.035m, γ=0.1 rad/m, $\omega = 628$ 1/s, $\lambda = 78$ GPa, $\mu = 44$ GPa, $\rho = 4370$ kg/m^3 (titanium alloy). Number of blades is N_0=32.

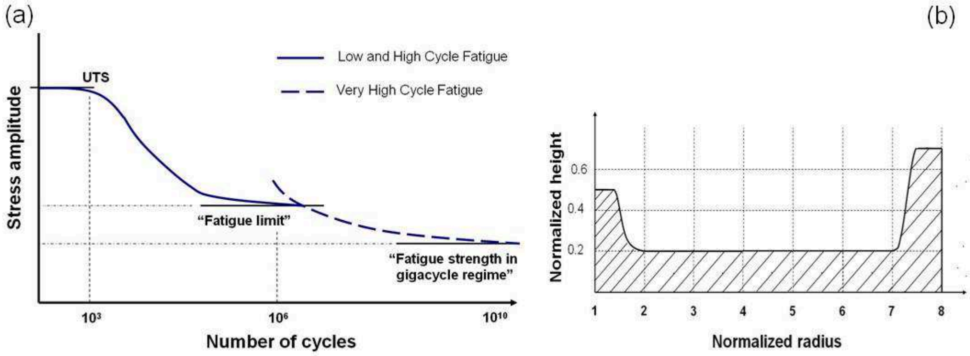

Fig. 1. Typical bimodal S-N curve (a) and a half-section of disk.

Radial distributions of stress components for two extreme positions (graphs (a) and (b)) of torsional cycles of blade vibration in the vicinity of the outer rim of the disc are shown in Fig. 2. In the notation of stress the indices 1,2,3 correspond to the coordinates r, θ, z.

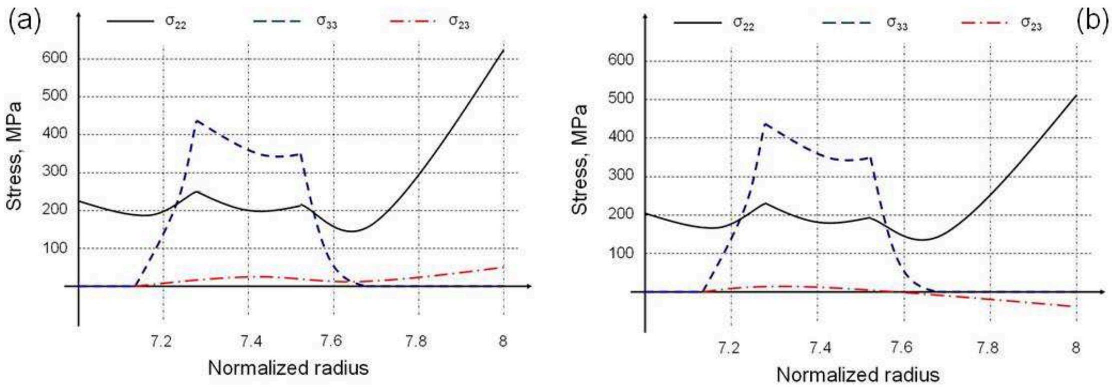

Fig. 2. The total radial distribution of stresses $\sigma_{22}, \sigma_{33}, \sigma_{23}$.

4. Zones of failure origin and estimation of service life duration in VHCF mode. Nowadays, there are no experimentally proved multiaxial criteria for VHCF. Therefore, to estimate the durability of compressor disk we used a well known Sines's criterion generalized for VHCF mode based on similarity between left and right branches of S-N curve, Fig. 1a. The parameters of multiaxial fatigue fracture model are determined based on uniaxial S-N curve. Generalization of uniaxial fatigue curve to multiaxial state is following (Sines' criterion) [2]:

$$\Delta\tau/2 + \alpha_s \sigma_{\text{mean}} = S_0 + AN^\beta, \sigma_{\text{mean}} = (\sigma_1 + \sigma_2 + \sigma_3)_{\text{mean}}, \ \Delta\tau = \sqrt{(\Delta\sigma_1 - \Delta\sigma_2)^2 + (\Delta\sigma_1 - \Delta\sigma_3)^2 + (\Delta\sigma_2 - \Delta\sigma_3)^2}/3$$

where σ_{mean} is a sum of principal stresses $\sigma_1, \sigma_2, \sigma_3$ averaged per loading cycle, $\Delta\tau$ is a change of shear stress per cycle. Parameters calculated [2] by using fatigue curves at $R = -1$ and $R = 0$ are:

$$S_0 = \sqrt{2}\sigma_u/3, \quad A = 10^{-3\beta}\sqrt{2}(\sigma_B - \sigma_u)/3, \quad \alpha_s = \sqrt{2}(2k_{-1} - 1)/3, \quad k_{-1} = \sigma_u/(2\sigma_{u0})$$

where σ_u and σ_{u0} are fatigue limits according to curves $\sigma_a(N)$ at $R = -1$ and $R = 0$ respectively, σ_B is the strength limit.

In order to account the similarity between the left and right branches of bimodal fatigue curves let's introduce substitutions $\sigma_B \to \sigma_u$, $\sigma_u \to \tilde{\sigma}_u$, $\sigma_{u0} \to \tilde{\sigma}_{u0}$, where $\tilde{\sigma}_u$ and $\tilde{\sigma}_{u0}$ are «new» fatigue limits for the right branch of fatigue curve at asymmetry coefficients $R=-1$ and $R=0$. As a result the generalized model parameters for VHCF mode may be written as [4]:

$$S_0=\sqrt{2}\tilde{\sigma}_u/3,\quad A=10^{-8\beta}\sqrt{2}(\sigma_u-\tilde{\sigma}_u)/3,\quad \alpha_s=\sqrt{2}(2k_{-1}-1)/3,\quad k_{-1}=\tilde{\sigma}_u/(2\tilde{\sigma}_{u0})$$

In VHCF calculations the following values of model parameters are used (titanium alloy): σ_u=350MPa, $\tilde{\sigma}_u$=250MPa, $\tilde{\sigma}_{u0}$=200MPa, β = -0.3.

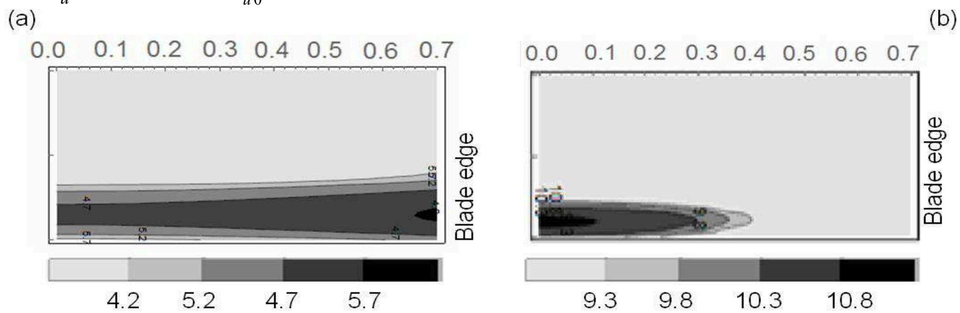

Fig. 3. Isolines of logarithm of durability in rectangular cross-section of contact zone at outer rim of disk under a blade.

Fig. 3 shows the distribution of logarithm of durability in rectangular contact zone between outer rim of the disc and a blade at $r=b$ (root cross-section of blade) for total LCF (a) and VHCF (b) modes. Dark color outlines critical regions of cyclic damage accumulation (zones of failure origin). For the given in section 3 loading parameters the LCF durability is estimated as $N_f=10^{4.2}$ that is equivalent to 40000-50000 flight hours (Fig. 3-a). Fig. 3-b outlines a critical region due to high frequency blades vibrations. This estimation gives the VHCF durability equal to $N_f=10^{9.3}-10^{10}$ cycles. The vibrations have a period of about 0.01s and the actual time to fatigue failure due to vibrations of the blades may reach a value of 20000 - 30000 hours. Therefore, high frequency vibration of blades can have a significant influence on the mechanism of fatigue crack initiation in real structures. Moreover, according to proposed estimation, the position of critical zone is also different for LCF and VHCF loading modes, Fig.3. In the case of LCF fatigue crack starts from the edge of the blade, while in VHCF mode the critical zone is found in the center of the blade with subsurface initiation.

Conclusions

Numerical calculations showed that vibrations of blades can lead to the high frequency cyclic stresses in compressor disk rim and cause crack initiation in VHCF regime. An estimation of fatigue durability due to LCF (flight cycles) and VHCF (blades vibrations) loads has shown a domination of VHCF mode of crack initiation (after 20000-30000 hours) over LCF mode (40000 – 50000 hours). Therefore, unexpected disk failures can be initiated by high frequency vibrations of the blades that are not usually taken into account by designers. The localization of critical zones is different for LCF and VHCF regimes. The VHCF lead to subsurface crack initiation.

Proposed model can be useful in express analysis of stress-states state of elements subjected to VHCF loading.

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