SUBSURFACE FATIGUE FRACTURE IN THE FAST-SPINNING REDUCTOR GEARWHEEL UNDER CONTACT LOADING

Nikitin I.S.[1]***, Burago N.G.***[1,2]***, Nikitin A.D.***[1, *]***, Stratula B.A.***[1]

[1] *Institute of computer-aided design of the Russian Academy of Sciences, 19/18, 2nd Brestskaya, Moscow, 123056, Russia.*
[2] *Institute for problems in mechanics of the Russian Academy of Sciences, 101, Pr. Vernadskogo, Moscow, 119526, Russia.*

[*] Alexander Nikitin, Doctor, ICAD RAS, 19/18 2nd Brestskaya, Moscow, 123056, Russia E-mail: Nikitin_alex@bk.ru,

Fatigue fracture in elements of turbo jet engine gear reduction is investigated under high angular rate loading conditions. The in-service fracture surface analysis, and numerical simulations were performed for a real aircraft structure. As an example, the driven bevel gear destruction of the main gearbox central drive of the PS90A engine is analyzed during operation. It is shown that the realized stress level in operation is close to the fatigue limit of the material. Therefore, with minor changes in the stress level of the gear, a significant scatter of durability can occur in the transition region between high-cycle fatigue (HCF) and very-high-cycle fatigue (VHCF). The numerical simulation is based on the developed by authors a multi-regime model of fatigue failure. The application of the proposed model of fatigue failure and the implemented through-calculation algorithm for calculating the damage function, allows to describe the main stages of the initiation and growth of fatigue quasi-cracks qualitatively and quantitatively during contact interaction in a rapidly rotating gearbox gear.

KEY WORDS: *fatigue, multi-regime model, numerical simulation, gearwheel, durability, damage function, crack initiation, steel.*

1. INTRODUCTION

Operating experience shows that fatigue crack appears in gears and its propagation can lead to the destruction of gears in operation at different in-service times (Korablev and Reshetov, 1968) – (Shanyavskiy and Skvortsov, 1999). The occurrence and propagation of cracks in-service is not allowed, since this can lead to a disruption in the functioning of the aircraft as a whole structure, for example, leading to engine failure due to loss of efficiency of the main gearbox.

The allowable stress level in gears, considering the margin of safety, is below the fatigue limit, which implies their operation for tens thousands of hours without destruction, if stress concentrators did not occur in them in the form of material defects at the production stage or damage did not appear during operation. This is confirmed by the experience of operating gears, in which the initiation and propagation of fatigue cracks took place during their long service life (Shanyavskiy and Skvortsov, 1999). At the time of the gear's destruction, when cracks initiated due to material tears and other defects, the gearwheel operating time is corresponding to more than 10^9 loading cycles. However, the number of cases with such crack initiation is small for this type of gear. So, even from defects in the material located on the surface of the gear, destruction occurs when the operating time is more than 10^9 loading cycles.

The operating time more than 10^9 single loading cycles exceeds the HCF area by an order of magnitude or even more in terms of operating time, for which the fatigue crack initiation and propagation laws have been studied and investigated in relation to various types of materials and, first, to iron-based alloys. In the case of an increase in the durability of gears, which are designed according to the principle of a safe resource, up to 10^9–10^{10} cycles, their limit state may occur with the initiation of fatigue cracks under the surface. Since the surface layer of gear wheels is subjected to saturation with carbon (cementing) to increase fatigue strength, this reduces the viscosity of the material, due to the difference in the plasticity of the layer and the core. Therefore, there is a high probability of natural fatigue crack initiation under the surface within the cemented layer during the increase in operating time (Korablev and Reshetov, 1968).

Achieving operating time of more than 10^9 loading cycles within the resource recommended by the designer in hours depends on the frequency of rotation of the gear wheels. Modern gearboxes use gears that rotate at speeds of more than 10,000 revolution per minute (rpm). This means that 10^9 loading cycles from the condition of a single loading of the wheel per revolution is achieved during the operation time of no more than 2000 hours. It is obvious that for long-term operation of gears for more than 10,000 hours, it is necessary to provide a stress level that corresponds to the durability of a smooth sample of at least 10^{10} cycles, which corresponds to the VHCF region for material operation in a structure.

One way or another, but for already designed and operating structures, the behavior of the material of which has not been studied in the region of VHCF and, therefore, the range of stress levels, in which destruction occurs by the VHCF mechanism, is unknown, and premature failure of parts due to crack appearance under the surface can take place. This is due to the discrepancy between the selected stress level and the recommended resource, which is limited due to the presence of inclusions in the material or due to the resulting (permissible) surface roughness in the technological cycle of its manufacture, when stress concentrators are underestimated. In the range of stresses corresponding to the transition from high-cycle fatigue to very high-cycle fatigue, there is a probability of crack initiation, both from the surface and under the surface of the material. The difference in the behavior of the material is due to the bimodal distribution of fatigue life in the stress range, which corresponds to a life of 10^7–10^9 cycles. In the indicated durability range at a fixed stress level, fatigue cracks can occur with different probabilities, both from the surface and under the surface, although the limit state is determined by the criterion of crack initiation from the surface. Therefore, in the indicated region of material failure, it is necessary to use not one, but two branches of the fatigue curve, i.e., to consider the bimodal distribution of fatigue life (Shanyavskiy et al., 2022), (Shanyavskiy, 2007).

2. REDUCTOR GEARWHEEL FATIGUE FRACTURE UNDER HIGH TO VERY HIGH CYCLE FATIGUE LOADING CONDITIONS

In cases of the gear wheels of gearboxes operating at high angular rate, a bimodal distribution of fatigue life can be observed with a wide spread of the operating time of the part in operation. So, for example, at the initial stage of operation of the PS-90A engine, there were cases of destruction of the gear wheel Z4 of the central drive of the main gearbox of the PS90A engine (Shanyavskiy et al., 2022), Fig. 1.

FIG. 1: View of the destroyed gear after 39 cycles of starting and stopping the gearbox.

According to the specifications, the gears are made of steel 20X3MVFA-Sh (EI415), nitrocarburized to the Rockwell hardness (HRC) of the working surface 59 HRC and core 34-43.5 HRC, polished to 6 degrees of accuracy.

The destruction of the wheels occurred from the surface from the geometric stress concentrators and under the surface of the parts. In this regard, it became necessary to establish the cause of their destruction in operation, to summarize the results of the studies performed and to give recommendations on how to eliminate the repetition of such cases.

Initially, all studied gears were subjected to metal-physical analysis to determine the conformity of the quality of the material in terms of chemical composition, structure, and mechanical characteristics, which are specified by the technical conditions for their manufacture. The material of the wheels is 20X3MVFA-Sh high-strength steel, nitrocarburized from the surface to a depth of 0.6 - 1.2 mm.

The performed spectral analysis of the gear wheel material showed that, according to the content of the main chemical elements - chromium, manganese, vanadium, molybdenum, and tungsten, it belongs to 20X3MVFA-Sh steel. Deviations in chemical composition from the specified grade of material, specified by the technical conditions, were not found in any of the investigated wheels.

The performed studies of the thickness of the nitrocarburized layer on transverse sections showed that it is in the range of 0.62 - 1.1 mm. This corresponds to the thickness of the carburized layer, given by the specifications for the manufacture of gears.

A metallographic study on transverse sections showed that the structure of the nitrocarburized layer within the teeth and the core of the part is satisfactory and corresponds to the state of the material for a given heat treatment, characterized by a material hardness ≥ 59 HRC.

Thus, based on a complex of metal-physical studies, it was shown that the material of the gears, the quality and depth of the nitrocarburized layer correspond to the technical conditions and, therefore, the destruction of the gears is not related to the state of their material.

Let us consider the regularities of occurrence of fatigue cracks in the Z4 gear (see Fig. 1). The dissipation of the operating time of the Z4 gear under consideration at the time of its destruction on the stand during the commissioning tests and in operation was 6-7285 hours. Provided that the rotational speed of the wheel in the operating mode is 14150 revolutions per minute (rpm), we find that at the time of the destruction of the wheels, the duration of their operation was in the range of operating time of about $5x10^6$ - $6x10^9$ cycles. This means that the gear wheels, at the lowest operating time, were destroyed in the HCF region, while at the maximum operating time, the destruction was realized in the VHCF region, although in all cases, with different operating hours, the crack initiation occurred from the surface of the part in the zone of scratches from machining.

Let us investigate one of the cases of wheel failure in operation, when during the flight, the PS-90A engine of the first power plant spontaneously turned off due to the destruction of the driven (Z4) bevel gear 93-06-079 (Shanyavskiy et al., 2022) (Fig. 2).

FIG. 2: View of the fracture of the gear wheel of the gearbox of the PS-90A engine.

The PS-90A engine under consideration has worked 92 hours or 37 engine start and stop cycles (ESSC) since the start of operation, i.e. when loading the gear wheel, $14150 x 60 x 92 = 7.8 x 10^7$ single material loading cycles were implemented. The indicated operating time corresponds to the transitional area from the HCF to the VHCF.

An assessment of the technical condition of the parts of the central drive showed that a fragment of a toothed rim, consisting of ten teeth, with part of the blade, separated from the wheel.

The fractographic study showed that the main fracture has a multicycle fatigue character (see Fig. 1) with crack initiation centers located on the surface of the interdental cavity from the side of the small module in the chamfer zone. The fracture surface in the entire area of fatigue crack development is characterized by the presence of mesolines, which are formed because of a change in the loading modes of the gear wheel during each flight cycle of its loading. As a result of counting the indicated regularly repeating mesolines, it was found that the destruction of the wheel continued for at least 28 flight cycles.

The use of the scanning electron microscope EVO 40 from Carl Zeiss showed that the initial crack initiation in the main direction of fracture most likely occurred from the surface of the radius transition, on which there are smoothed marks from machining (Fig. 3). One of the scratches could serve as a stress concentrator for an incipient crack.

FIG. 3: Type of scratches in the gear recess, from the base of which the initiation and development of a crack is realized.

The high-cycle fatigue failure of the wheel should not be associated with a high stress concentration due to the identified risks near the crack initiation site but should be attributed to the high stress of the part in this zone. The relatively low running time of the wheel (92 hours, 37 engine’s start and stop cycles (ESSC)) compared to the rest of the gear wheels operating without accidents indicates the following. The realizable level of stress in operation is close to the fatigue limit of the material. Therefore, with minor changes in the tension level of the gear wheel, which is associated, among other things, with the appearance of stress concentrators in the form of scratches from machining, it can cause a dissipation of durability by almost three orders of magnitude in the region of transition from HCF to VHCF.

In conclusion of this section, it should be emphasized that reducing the stress concentration on the surface will certainly have a positive effect of increasing the life of the Z4 gear, but this will not solve the problem, because. when running more than 10,000 hours, wheel failures are still observed, and their intensity will increase. This is because in terms of tension it is in the transition region of the HCF-

VHCF. A decrease in surface concentration, the absence of cementite inclusions near the surface will lead to a more pronounced initiation of fatigue cracks under the surface.

3. MULTI REGIME TWO-CRITERION MODEL OF FATIGUE FAILURE

For mathematical modeling of the process of fatigue failure of the gear wheels of the gearbox and evaluation of the service life, we will use the scheme for describing the full fatigue curve based on the Baskin relation (Basquin, 1910), which establishes the relationship between the stress level and the number of cycles to failure:

$$\sigma_a = \sigma_u + \sigma_c N^{-\beta} \tag{1}$$

where σ_a is the amplitude of the cyclic load, N is the number of cycles to failure, σ_u is the fatigue limit, σ_c is the power-law coefficient, β is an indicator describing the slope of the fatigue curve. An analysis of the experimental fatigue curves shows that the nature of the drop in cyclic strength with increasing loading cycles is similar for the left (LCF, HCF) and right (VHCF) branches, which allows us to formulate a hypothesis about the similarity of their description (Burago et al., 2016). Within the framework of this model, it is assumed that the left and right branches of the complete fatigue curve are described by a Baskin-type relation (1). To distinguish between parameters and variables related to different branches, we introduce indices: L - for the left branch and V - for the right one. In this case, the ratios for the left and right branches take the following form:

$$\sigma_{eq} = \sigma_u + \sigma_L N^{-\beta_L}, \quad \sigma_{eq} = \tilde{\sigma}_u + \sigma_V N^{-\beta_V} \tag{2}$$

Here σ_u is the classical fatigue limit, $\tilde{\sigma}_u$ is the fatigue limit in the VHCF region, σ_{eq} is the equivalent stress, which for the case of uniaxial loading coincides with the amplitude of the cyclic load.

The parameters for the left and right branches can be determined from the results of the corresponding experimental tests for uniaxial loading according to the method described in [7], based on the analysis of the complete fatigue curves. So, at high levels of external load, when the amplitude of cyclic loading can exceed the yield strength of the material, the fatigue strength differs slightly from the tensile strength of the material σ_B. Depending on the material, this trend is observed in the first 10^2–10^3 loading cycles. Thus, the condition for the left branch will be the exit of the curve to the ultimate strength of the material on a given test base (for definiteness, we will take 10^3 cycles). Substituting the tensile strength instead of the amplitude of the cyclic load, we can obtain an expression for σ_L. Carrying out similar reasoning for the right branch of the complete fatigue curve, it can be noted that it acquires a descending character at an external load level of the order of the classical fatigue limit after 10^7–10^8 cycles. Taking for definiteness the test base of 10^8 cycles, we can obtain an expression for the parameter σ_V. Thus, the parameters of the generalized Baskin-type relation are determined by the quasi-static and fatigue strength characteristics of the material:

$$\sigma_L = 10^{3\beta_L}(\sigma_B - \sigma_u), \qquad \sigma_V = 10^{8\beta_V}(\sigma_u - \tilde{\sigma}_u) \tag{3}$$

In the case of a complex stress-strain state, the equivalent stress can be determined in accordance with experimentally validated multiaxial fatigue failure criteria. In addition, equivalent stresses can be used to describe various opening mechanisms: microcracks of normal opening or shear. If several criteria are simultaneously used for a material that describe the fatigue strength of normal and shear opening, then the model allows one to determine the type of opening in the nucleation region. In the present work, two criteria are chosen: Smith-Watson-Topper (SWT) (normal opening microcrack mechanism)

(Smith et al., 1970) in the form (Gates and Fatemi, 2016) and Carpinteri-Spagnoli-Vantadori (CSV) (shear microcrack mechanism) (Carpinteri et al., 2011).

Fracture by normal opening mechanisms is associated with tensile stress components. In the SWT criterion in the form [9], a variant of taking into account these stress components is presented:

$$\sigma_{eq} = \sigma^{n} = \sqrt{\left\langle \sigma_{1_{\max}} \right\rangle \Delta\sigma_{1} / 2} \tag{4}$$

where $\left\langle \sigma_{1_{\max}} \right\rangle$ is the value of the maximum main tensile stress, $\left\langle \sigma_{1_{\max}} \right\rangle = \sigma_{1_{\max}} H(\sigma_{1_{\max}})$, $\Delta\sigma_{1} / 2$ is the amplitude of the main cyclic stress. To describe the development of shear-type microcracks, a criterion is used that considers the features of their formation. It often happens that shear stresses alone are not enough to form a crack, and the most dangerous areas are those where normal and shear stresses act simultaneously. One of the criteria to effectively describe such cracks is the CSV criterion (Carpinteri et al., 2011).:

$$\sigma_{eq} = \sigma^{\tau} = \sqrt{(\left\langle \Delta\sigma_{n} \right\rangle / 2)^{2} + 3(\Delta\tau_{n} / 2)^{2}} \tag{5}$$

where $\Delta\tau_{n} / 2$ is the amplitude of the maximum shear stress acting on a certain area with normal *n*, $\left\langle \Delta\sigma_{n} \right\rangle / 2$ is the amplitude of the cyclic stress in the stretching phase $\left\langle \Delta\sigma_{n} \right\rangle = \sigma_{n_{\max}} H(\sigma_{n_{\max}}) - \sigma_{n_{\min}} H(\sigma_{n_{\min}})$ acting on the same area. The coefficients in front of these amplitudes indicate the dominant role of shear stresses in fracture. To determine the failure mechanism for a given configuration of external loads, it is necessary to check according to two criteria for one stress state: $\sigma_{eq} = \max(\sigma^{n}, \sigma^{\tau})$

Under cyclic loading, there is a gradual degradation of the properties of the material, the formation and growth of microcracks, which leads to a continuous change in the stress state. For a correct description and modeling of the process of fatigue failure, it is necessary to consider the kinetics of these processes.

To describe the cyclic degradation of a material, the concept of a distributed damage function ψ is introduced, which takes values from 0 to 1 (Rabotnov, 1959), (Kachanov, 1958), Lemaitre and Chaboche, 1994), (Murakami, 2012) and is conventionally equal to the relative density of microdefects in a small volume of a deformable sample. For an undamaged material particle, the value of the function ψ is 0, and for a destroyed one, it is 1. The change in the damage function with increasing loading cycles is described by the kinetic equation proposed in (Nikitin et al., 2020):

$$d\psi / dN = B(\sigma, \Delta\sigma)\psi^{\gamma} / (1 - \psi^{1-\gamma}) \tag{6}$$

Where $B(\sigma, \Delta\sigma)$ is the coefficient depending on the stress state in the cycle, $\Delta\sigma$ is the range of the cyclic load, γ is an experimentally determined parameter describing the rate of damage accumulation. Expressions for the coefficients $B(\sigma, \Delta\sigma)$ can be obtained by comparing the representations of uniaxial fatigue curves (2) for a symmetric tension-compression cycle with the solution of the kinetic equation for the damage function in a uniform stress state by integrating within the range from 0 to 1 (Nikitin et al., 2020). A generalization of the expressions for the coefficients, which considers the possibility of implementing two mechanisms for the development of microcracks, is presented in (Nikitin et al., 2021), (Nikitin et al., 2022). As a result, for the left and right branches of the fatigue curve, the analytical expressions for the coefficients are:

$$\begin{aligned} B = B_{L} &= 10^{-3} \left[\left\langle \sigma_{eq} - \sigma_{u} \right\rangle / (\sigma_{B} - \sigma_{u}) \right]^{1/\beta_{L}} / (1-\gamma) / 2 \qquad \text{for } \sigma_{u} + \Delta\sigma_{u} < \sigma_{eq} < \sigma_{B} \qquad (7) \\ B = B_{V} &= 10^{-8} \left[\left\langle \sigma_{eq} - \tilde{\sigma}_{u} \right\rangle / (\sigma_{u} - \tilde{\sigma}_{u}) \right]^{1/\beta_{V}} / (1-\gamma) / 2 \qquad \text{for } \tilde{\sigma}_{u} < \sigma_{eq} \le \sigma_{u} + \Delta\sigma_{u} \end{aligned}$$

where $\Delta\sigma_u = 10^{-5\beta_L}(\sigma_B - \sigma_u)$ is the width of the bifurcation region. The expressions in triangular brackets are “markers” of the fatigue curve branch and are defined as follows $\langle f \rangle = fH(f)$, where $H(f)$ is the Heaviside function. The exponent γ varies in the range $0 < \gamma < 1$.

The development of damage in a material particle leads to an effective decrease in the elastic moduli, in the general case according to a nonlinear law, and in the proposed version of the model, according to a piecewise linear law of the following form.

Material degradation at $\psi < \psi_*$, $\lambda(\psi) = \lambda_0(1-\kappa\psi)$, $\mu(\psi) = \mu_0(1-\kappa\psi)$.

Complete destruction at $\psi_* \le \psi \le 1$, $\lambda = 0$, $\mu = 0$.

Here, $\psi_* \lesssim 1$ is the critical value of damage at which a state of complete destruction occurs. The value of the critical value of the damage function is slightly less than 1.

4. NUMERICAL METHOD FOR DAMAGE FUNCTION CALCULATION

The numerical method for calculating damage zones consists in step-by-step (by loading cycles) calculation of the elastic stress-strain state of a material sample or structural element, in parallel with the numerical solution of the nonlinear equation for damage (6) and the correction of the elastic moduli of the medium in areas where the damage function is different from zero. Such areas become additional developing stress concentrators, and narrow extended zones of complete destruction in the above sense will be called "quasi-cracks".

Difference approximation of equation (6) can be performed by direct integration over the interval of two discrete values of cycles N^n and N^{n+1}, which makes it possible to obtain an analytical solution for the values ψ_k^{n+1} of the damage function at each spatial grid node with a given time step (number of cycles)

$$\psi_k^{n+1} = \left(1 - \sqrt{\left(1-(\psi_k^n)^{1-\gamma}\right)^2 - 2(1-\gamma)B^n \Delta N^n}\right)^{1/(1-\gamma)} \tag{8}$$

Where ψ_k^{n+1} is the value of the damage function at the *k*-th spatial node on *n+1*-time layers, $\Delta N^n = N^{n+1} - N^n$ is the step by loading cycles.

To determine the global calculation step in terms of the number of cycles for the entire sample, by enumerating the grid nodes, one is selected in which, in the current stressed state, the local step to achieve the critical value of the damage function is minimal. The required step value is considered equal to half of the minimum local one (Nikitin et al., 2021):

$$\Delta N^n = \min_k 0.5\Delta\tilde{N}_k^n, \quad \Delta\tilde{N}_k^n = \left[\psi^{1-\gamma}/(1-\gamma) - \psi^{2(1-\gamma)}/2/(1-\gamma)\right]\Big|_{\psi_k^n}^{1} / B^n \tag{9}$$

The relationship between the values of elastic characteristics and damage functions is taken in the following form (Nikitin et al., 2020) - (Nikitin et al., 2022)

$$E_k^{n+1} = E_0\left(1-\kappa\psi_k^{n+1}\right)\left(H\left(\psi_* - \psi_k^{n+1}\right) + 0.001\right) \tag{10}$$

Where E_k^{n+1} is the value of the elastic modulus at the new step, E_0 is the Young's modulus of elasticity of the undamaged material, κ is the degradation coefficient of the modulus, which is established in the course of computational experiments. The Poisson's ratio of the material is assumed to be constant.

In the numerical method, it is assumed that in the state of complete destruction, the material has minimal residual moduli of elasticity, conditionally equal to 0.001 of its initial value. This makes it possible to carry out calculations on a fixed grid, solving a strongly inhomogeneous elastic problem at each step over loading cycles.

5. DEVELOPMENT OF FATIGUE QUASI CRACKS IN THE REDUCTOR GEARWHEEL

Based on the proposed method, mathematical modeling of the process of initiation and development of a fatigue quasi-crack in a rapidly rotating gear during contact interaction of teeth was carried out. Gears material - steel 20X3MVFA-Sh.

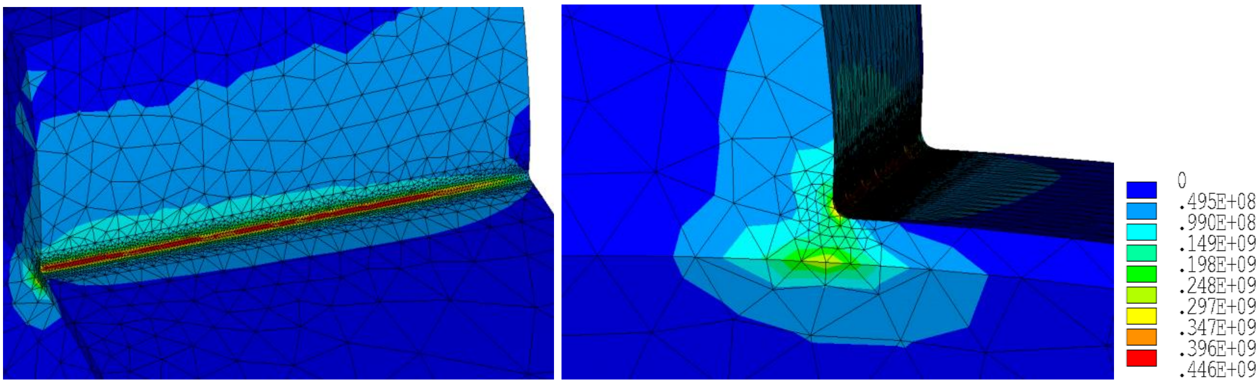
The steel density is equal to ρ =7800 kg/m^3, Young's modulus E=207 GPa. As already mentioned, the near-surface layer is 1.2 mm nitrocarburized and has a hardness 59 HRC. The hardness of the core lies in the range 34-41 HRC (Shanyavskiy et al., 2022), for definiteness, we choose the average value of 38. The reference limits of strength and fatigue of the wheel core are σ_B=910 MPa, σ_u=400 MPa. The characteristic value of the fatigue limit of the VHCF, as a rule, is 20-30% lower than the classical fatigue limit, so we will take the value $\tilde{\sigma}_u$=280 MPa.

There are no data on the strength properties of the nitrocarburized material, so we will make the following estimates. Since the strength properties of materials are strongly correlated with their Rockwell hardness, to determine the strength characteristics of a nitrocarburized near-surface layer, we multiply the standard values for this alloy by a factor of 59/38.

Then for the strength properties of the nitrocarburized near-surface layer to a depth of 1.2 mm, the following values can be taken: σ_B=1420 MPa, σ_u=620 MPa, $\tilde{\sigma}_u$=420 MPa. In the calculation scheme, one gear wheel is rigidly fixed on the axle, a moment of 420 Nm is applied to the second gear from the side of the axle. Cyclic loading in the form of stationary vibrations with a frequency of 14100 rpm=235 Hz is considered. For contact interaction, friction is considered with a coefficient of 0.2.

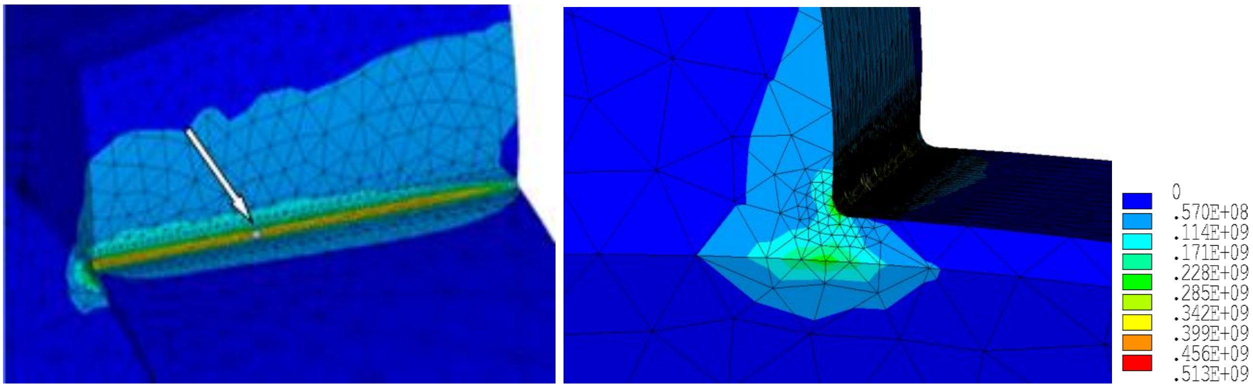
Elastic calculations of the loading cycle in dynamic mode (high-frequency oscillations, VHCF) were performed using the ANSYS software package, supplemented with a code for calculating the kinetics of fatigue damage and changing the elastic moduli in accordance with the above algorithm.

Fig. 4 shows the computational grid and fields of equivalent stresses at the beginning of loading before the development of damage. Fig. 5 shows the equivalent stresses according to the implemented SWT criterion at the tooth base at the moment of quasi-crack initiation (gray dot in the middle of the tooth base). This moment corresponds to the number of cycles N=5.4 x10^7 cycles.



(a) (b)

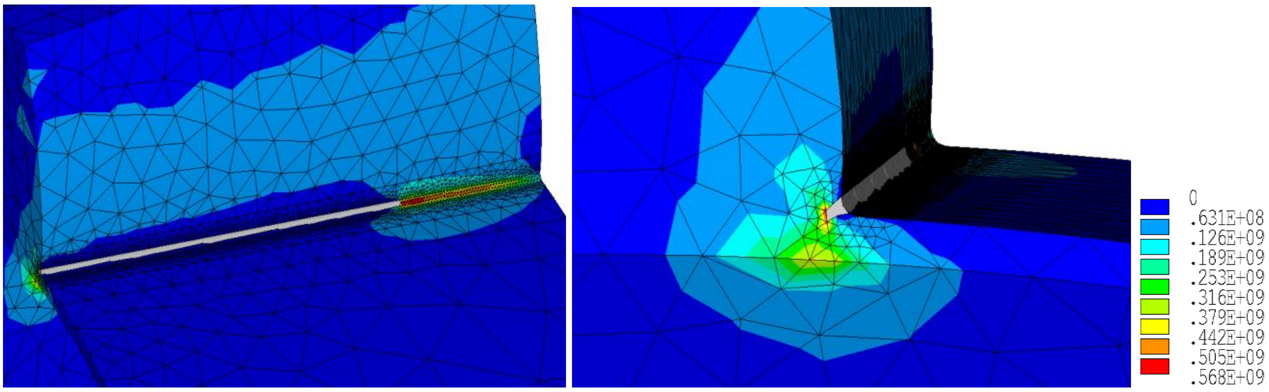
FIG. 4: Equivalent stresses according to the SWT criterion. Start of loading



(a) (b)

FIG. 5: Equivalent stresses according to the SWT criterion at the base of the tooth at the moment of quasi-crack initiation, *N*=5.4 x10^7 cycles.

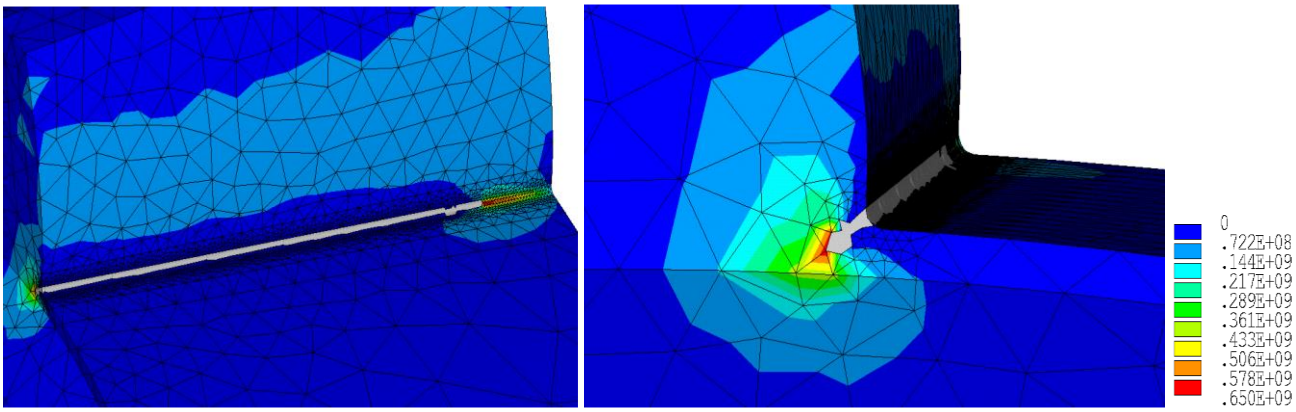
As the fatigue process develops, a quasi-crack grows at the base of the tooth and exits to the end surface at the moment *N*=1.8x10^8 (Fig. 6).



(a) (b)

FIG. 6: Crack exit to the end surface, *N*=1.8x10^8 cycles.

Then the quasi-crack begins to grow deep from the surface of the radius transition (Fig. 7, *N*=3.1x10^8), this is the final stage of the process, which leads to macro-fracture of the gear. The obtained value of durability coincides in order of magnitude with the number of cycles before failure in operation, given above (7.8x10^7) in the description of a specific flight accident. In any case, this calculation confirms the possibility of realization of fatigue failure by the VHCF mechanism, especially considering the potential dissipation of durability in the region of transition from high-cycle to very-high-cycle fatigue.



(a) (b)

FIG. 7: Further growth of the crack in depth from the surface of the radius transition, *N*=3.1x10^8 cycles.

Thus, with the help of the proposed multi-mode model of fatigue failure and the numerical calculation algorithm based on it, it was possible to qualitatively and to a certain extent quantitatively describe the main stages of the process of super-high-cycle failure during contact interaction in a rapidly rotating gear transmission of a gearbox described in the first section.

6. CONCLUSIONS

Fatigue failure of fast-rotating elements of the gearbox of a gas turbine aircraft engine has been studied under very-high-cyclic loading. As an example, the case of destruction of the driven bevel gear of the central drive of the main gearbox of the PS90A engine during operation is analyzed. It is shown that the realized stress level in operation is close to the fatigue limit of the material. Therefore, with minor changes in the tension level of the gear, a significant dissipation of durability can occur in the transition region from high-cycle to very-high-cycle fatigue.

A description is given of the previously proposed model of fatigue failure based on the damage theory, which makes it possible to obtain estimates of the fatigue life of a material in a wide range of loading cycles. The model includes two mechanisms for the initiation of fatigue cracks associated with normal opening and shear. The two-criterion model makes it possible to predict the type of crack opening in the nucleation region, which is a distinctive feature of this model from several other approaches. A unified description of the right and left branches of the fatigue curve makes it possible to present a unified algorithm for calculating cyclic damage with automatic determination of the parameters of the kinetic equation depending on the stress-strain state. The proposed model was used to assess the fatigue life of the VHCF destruction of the gear wheels of the gearbox in operation.

With the help of the implemented algorithm for calculating the damage function in a high-frequency cyclic process, it is possible to qualitatively and to a certain extent quantitatively describe the main stages of the initiation and growth of fatigue quasi-cracks during contact interaction in a rapidly rotating gearbox gear.

ACKNOLEDGMENTS

The work is realized in the framework of the Russian Science Foundation project No. 19-19-00705-P.

REFERENCES

Basquin O.H. The exponential law of endurance tests// Proc. of the American society for testing and material. 1910. Vol.10. Pp. 625–30.

Burago N.G., Zhuravlev A.B., Nikitin I.S., Yakushev V.L. A study of different modes of fatigue fracture and durability estimation for compressor disks of gas-turbine engine// Mathematical Models and Computer Simulations. 2016. Vol. 8. № 5. Pp. 523-532.

Carpinteri, A., Spagnoli, A. and Vantadori, S., 2011. Multiaxial assessment using a simplified critical plane based criterion// Int. J. of Fatigue. 2011. Vol. 33. Pp. 969-76.

Gates N., Fatemi A. Multiaxial variable amplitude fatigue life analysis including notch effects// Int. J. of fatigue. 2016. Vol. 91. Pp. 337-351.

Kachanov L.M. On the time of fracture under creep loading// News. AH USSR OTH. 1958. 8. Pp. 26-31.

Korablev A.I., Reshetov D.N. Increasing the bearing capacity and durability of gears. -M.: Mashinostroenie. 1968.

Lemaitre J. Chaboche J.L. Mechanics of solid materials. Cambridge: Cambridge University Press. 1994. 584 p.

Murakami S. Continuum Damage Mechanics. A Continuum Mechanics Approach to the Analysis of Damage and Fracture. Dordrecht: Springer. 2012.

Nikitin I.S., Burago N.G., Zhuravlev A.B. and Nikitin A.D. Multi-mode model for fatigue damage development// Mechanics of Solids. 2020. Vol. 55. No 8. Pp. 298–306.

Nikitin I.S., Burago N.G., Nikitin A.D., Stratula B.A. Multi-mode Model and Calculation Method for Fatigue Damage Development. In: Jain L.C., Favorskaya M.N., Nikitin I.S., Reviznikov D.L. (eds.) Applied Mathematics and Computational Mechanics for Smart Applications. Smart Innovation, Systems and Technologies. 2021. Vol 217. Springer, Singapore.

Nikitin I.S., Nikitin A.D. Multi regime model and numerical algorithm for calculations on various types quasi crack developing under cyclic loading// Computer research and modeling. 2022. Vol. 14. No 4. Pp. 871-883.

Rabotnov Y.N. On mechanisms of fatigue failure. Issues of strength of materials and structures // News. AH USSR OTH. 1959. Pp. 5-7.

Shanyavskiy A.A., Skvortsov G.V. Crack growth in the gigacycle fatigue regime for helicopter gears// Fatigue Fract. Engng. Mater. Struct. 1999. V.22. Pp. 609-619.

Shanyavskiy A.A. Modeling of fatigue failures in metals. -Ufa: Monographiya. 2007.

Shanyavskiy A.A., Tushencov A.L., Soldatenkova M.A. Very high cycle fatigue of the conic gear wheel central drive and gearbox of PS-90A engines// Proceedings of the ORAP. Moscow. 2017. № 29. Pp. 210-217.

Shanyavskiy A.A., Nikitin A.D., Soldatenkov A.N. Very high cycle fatigue of metals.. Synergetics and physical mesomechanics. - M.: Physmatlit. 2022. 496 p.

Smith R.N., Watson P., Topper T.H. A stress-strain parameter for the fatigue of metals// J. of Materials. 1970. Vol. 5. Pp. 767-78.