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Explicit-Implicit schemes for solving the problems of the dynamics of isotropic and anisotropic elastoviscoplastic media

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Abstract. A method is developed for the numerical solution of dynamic elastic-visco-plasticity equations for isotropic and anisotropic cases. Anisotropic elastic-visco-plastic systems of equations often describe models of deformable solid media with a discrete set of slip planes (in layered and block media) with nonlinear slip conditions at contact boundaries. For a stable numerical solution of such systems of differential equations, an explicitly implicit method is proposed. An explicit approximation is used for the equations of motion while an implicit approximation is applied to the constitutive relations containing a small parameter in the denominator of the free term. Examples of numerical solutions are presented for dynamic problems on the reflection of waves from obstacles in the inelastic layered and block media with the formation of slip and delamination zones at the boundaries of structural elements.

1. Introduction

Continuous models of deformable solid media with a discrete set of slip planes (in layered and block media) with nonlinear (viscoplastic) slip conditions at contact boundaries can be obtained by asymptotic homogenization [1] or using a discrete version of the slip theory [2]. In all these cases, the constitutive relations contain a nonlinear free terms and have a short relaxation time of the stresses (hence, a small parameter in the denominator of the free term). For a stable numerical solution of such system of differential equations, an explicitly implicit method with an explicit approximation of the equations of motion and an implicit approximation of the constitutive relations containing a small parameter in the denominator of the free term is proposed. Nonlinear algebraic equations of implicit approximation are solved analytically by method of expansions in powers of a small parameter. Formulas of the first and second order of accuracy for correcting stresses after the "elastic" step of calculating are obtained. These formulas admit a limiting transition to the zero value of the small parameter (relaxation time) of the initial differential system of equations, which corresponds to the limiting transition from an elastic-visco-plastic model of a continuous medium to an elastoplastic one.

Examples of numerical solutions of dynamic problems on the reflection of waves from obstacles in a layered and block medium with formation of slip zones and delamination at the boundaries of structural elements are presented.

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2. Nonlinear conditions on the contact boundaries of structural elements

In a Cartesian rectangular coordinate system x_i $(i=1,2,3)$ consider an infinite elastic medium with an oriented system of periodically repeating parallel slip planes. The orientation of this system is given by the unit normal $\mathbf{n}$. The distance between the slip planes is constant and equal ε. The density of the material ρ, as well as the Lamé elastic moduli λ and μ, are considered to be given constants. The stress state is described by the stress tensor $\boldsymbol{\sigma}$. The tangential stress vector on the slip plane is $\boldsymbol{\tau}=\boldsymbol{\sigma}\cdot\mathbf{n}-(\mathbf{n}\cdot\boldsymbol{\sigma}\cdot\mathbf{n})\mathbf{n}$, the normal stress is $\sigma_n=\mathbf{n}\cdot\boldsymbol{\sigma}\cdot\mathbf{n}$. We introduce the vectors of shear $\boldsymbol{\gamma}$ and detachment $\boldsymbol{\omega}=\omega\mathbf{n}$ velocities, determined by jumps in the tangent $[\mathbf{V}_\tau]$ and normal $[V_n]$ velocities at the contact boundaries:

$$\boldsymbol{\gamma}=[\mathbf{V}_\tau]/\varepsilon,\ \omega=[V_n]/\varepsilon.$$

It is assumed that physically between the elastic layers there are thin viscoplastic interlayers of thickness $\delta<<\varepsilon$, but we neglect the thickness of these interlayers and replace them by sliding conditions on the contact boundaries of the layers. We take the conditions of contact interaction in the following form. Nonlinear viscoplastic slip occurs when $\sigma_n<0$:

$$\boldsymbol{\gamma}=\kappa\boldsymbol{\tau}<F\left(|\boldsymbol{\tau}|^2/\tau_s^2-1\right)>,\ \ \omega=0$$

The detachment occurs when $\Omega\geq 0$, $\boldsymbol{\tau}=\sigma_n=0$. Here $\Omega=[u_n]/\varepsilon$ is the normalized jump of the normal displacements at the contact boundary, determined by the equation $\dot{\Omega}=\omega, \kappa=\delta/(\varepsilon\eta)$, η is the viscosity coefficient, $<F(y)>=F(y)H(y)$ is a nonlinear function different from zero beyond the yield limit $|\boldsymbol{\tau}|=\tau_s$, $H(y)$ is the Heaviside function, $H(y)=0$ for $y<0$, $H(y)=1$ for $y\geq 0$. Here and below the dot above the symbol denotes the time derivative ($\dot{\Omega}=d\Omega/dt$). If $<F(y)>=1$ then the nonlinear condition of viscoplastic sliding is transformed into a linear condition for viscous sliding. The contact plane with the indicated interaction conditions will be called the slip plane.

In order to pass to the continuum model of a medium containing a system of such slip-peel planes, we will also consider $\boldsymbol{\gamma}$ and $\boldsymbol{\omega}$ both as continuous functions of coordinates and time. These functions have the meaning of distributed velocities of slip and delamination and are used in the same manner as in constructing models of inelastic media with a continuous distribution of slip planes (see the review [2]). These relationships make it possible to take into account the contribution of the velocities of slip $\boldsymbol{\gamma}$ and delamination $\boldsymbol{\omega}$ to the strain tensors of inelastic deformation $\mathbf{e}^\gamma$ and $\mathbf{e}^\omega$, respectively:

$$\mathbf{e}^\gamma=(\mathbf{n}\otimes\boldsymbol{\gamma}+\boldsymbol{\gamma}\otimes\mathbf{n})/2\ ,\ \ \mathbf{e}^\omega=(\mathbf{n}\otimes\boldsymbol{\omega}+\boldsymbol{\omega}\otimes\mathbf{n})/2=\omega\mathbf{n}\otimes\mathbf{n}$$

The strain rate tensor is obtained by summing all the elastic and inelastic components:

$$\mathbf{e}=\mathbf{e}^e+\mathbf{e}^\gamma+\mathbf{e}^\omega,\ \mathbf{e}=(\nabla\mathbf{v}+\nabla\mathbf{v}^T)/2$$

Here $\mathbf{v}$ is the "macroscopic" velocity of the particles of the medium, $\mathbf{e}^e$ is the elastic strain rate tensor, which is related to the stress tensor by Hooke's law: $\dot{\boldsymbol{\sigma}}=\lambda(\mathbf{e}^e:\mathbf{I})\mathbf{I}+2\mu\mathbf{e}^e$

The continuous conditions for $\boldsymbol{\gamma}$ and $\boldsymbol{\omega}$, corresponding to local contact conditions, have the form:

$$\boldsymbol{\gamma}=\kappa\boldsymbol{\tau}<F\left(|\boldsymbol{\tau}|^2/\tau_s^2-1\right)>,\ \omega=0\ \text{ for }\ \sigma_n<0$$

$$\boldsymbol{\tau}=\sigma_n=0\ \text{ for }\ \Omega\geq 0$$

The system is closed by the equations of motion: $\rho\dot{\mathbf{v}} = \nabla \cdot \boldsymbol{\sigma}$

3. Continuous models

3.1. Model of a layered medium.
In a layered medium consisting of elastic layers, there is a single system of slip-slip planes with the normal $\mathbf{n}$, its structure is shown schematically in a two-dimensional version in figure 1-a. The selected contact conditions are satisfied at the boundaries of the layers.

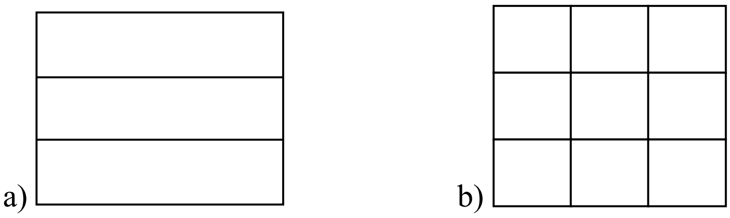


Figure 1. a) layered medium, b) block medium.

If the direction of the normal to the slip-delamination plane coincides with the direction of the axis x_2 of the adopted coordinate system, then for the normal the relation $n_j = \delta_j^2$ is valid, where δ_j^i is the Kronecker symbol. Using Heaviside functions $\tilde{H}(x) = 1 - H(-x)$ in the case $n_j = \delta_j^2$, the system of equations of a layered medium can be written in a visual form, clearly isolating the equations for each desired function::

$$\rho\dot{v}_i = \sigma_{ij,j}\,,\ \dot{\sigma}_{ii} \underset{i\neq 2}{=} \lambda v_{k,k} + 2\mu v_{i,i} - \lambda\omega\,,\ \dot{\sigma}_{22} = (\lambda v_{k,k} + 2\mu v_{2,2})\tilde{H}(-\sigma_{22})$$

$$\dot{\sigma}_{ij} \underset{i,j\neq 2}{=} \mu(v_{i,j} + v_{j,i})\,,\ i \neq j\,,\ \dot{\sigma}_{2j} \underset{j\neq 2}{=} \left[\mu(v_{2,j} + v_{j,2}) - \mu\gamma_j\right]\tilde{H}(-\sigma_{22})$$

$$\gamma_j = \kappa\sigma_{2j} < F\left(\left|\boldsymbol{\tau}\right|^2 / \tau_s^{\ 2} - 1\right) > \tilde{H}(-\sigma_{22}) + (v_{2,j} + v_{j,2})H(\sigma_{22})H(\Omega)$$

$$\omega = (\lambda v_{k,k} + 2\mu v_{2,2})/(\lambda + 2\mu)\,H(\sigma_{22})H(\Omega)\,,\ \dot{\Omega} = \omega\,,\ i,j = 1,2,3\,,\ \left|\boldsymbol{\tau}\right| = \sqrt{\sum_{k\neq 2}\sigma_{2k}\sigma_{2k}}$$

The resulting system of equations must be supplemented by conditions on the boundary Γ of the region occupied by the medium: $\boldsymbol{\sigma}\cdot\mathbf{n}\big|_{\Gamma} = \mathbf{f}_{\Gamma}$ or $\mathbf{v}\big|_{\Gamma} = \mathbf{v}_{\Gamma}$, as well as by the initial conditions for $t = 0$: $\boldsymbol{\sigma} = \mathbf{v} = \Omega = 0$.

In the slip regime for $\sigma_{22} < 0$, this system is a semilinear hyperbolic system of equations with a small parameter in the denominator of the free term, which describes an anisotropic elastic-viscoplastic medium.

3.2. Model of a block medium.
Block medium is formed by uniformly stacked elastic solid cubes (parallelepipeds) with three possible slip and delamination planes oriented by mutually perpendicular unit normals $\mathbf{n}^{(s)}$, $s = 1,2,3$. Schematically, its two-dimensional version is shown in figure 1-b.

The vectors of the velocity of sliding and delamination in the plane with the normal $\mathbf{n}^{(s)}$ is denoted by $\boldsymbol{\gamma}^{(s)}$ and $\boldsymbol{\omega}^{(s)}$. We also denote the component $\boldsymbol{\gamma}^{(s)}$ in the direction $\mathbf{n}^{(i)}, s \neq i$ as $\gamma_i^{(s)}$. We will also call the slip-delamination plane with the normal $\mathbf{n}^{(s)}$ as s-plane. The picture of possible slip on contact surfaces is described as follows.

If there is a slip on the s-plane and $\gamma_i^{(s)} \neq 0$, $\gamma_j^{(s)} \neq 0$, $i \neq j$, then the regularity of the block medium structure is violated, because due to overlap of i- and j-planes, they cease to be planes of possible sliding, and become the planes of possible delamination with contact conditions of the following form:

$$\gamma_i^{(s)} = 0,\ \omega^{(s)} = 0 \text{ for } \sigma_n^{(s)} < 0$$

$$\sigma_n^{(s)} = 0,\ \gamma_i^{(s)} = 0 \text{ for } \Omega^{(s)} \geq 0$$

Thus, it is assumed that only one of the three possible slip and delamination planes, depending on the type of stress state, is realized. The remaining two planes become planes of delamination.

If we orient the three normals of the possible slip and delamination planes in accordance with the coordinate axes of the adopted coordinate system, then for the normals the following relation will hold: $n_i^{(s)} = \delta_i^s$, where δ_i^s is the Kronecker symbol. As in the case of a layered medium, we get the system of equations for a block medium in a form convenient for application (here j_s-plane is the slip plane):

$$\rho \dot{v}_i = \sigma_{ij,j},\ \dot{\sigma}_{jj} = \left(\lambda v_{k,k} + 2\mu v_{j,j} - \lambda \sum_{l \neq j} \omega^{(l)} \right) \tilde{H}(-\sigma_{jj})$$

$$\dot{\sigma}_{ij} = \mu(v_{i,j} + v_{j,i}),\ i, j \neq j_s,\ i \neq j$$

$$\dot{\sigma}_{ij} = \left[\mu(v_{i,j} + v_{j,i}) - \mu(\gamma_i^{(j)} + \gamma_j^{(i)}) \right] \tilde{H}(-\sigma_{jj}),\ j = j_s,\ i \neq j$$

$$\gamma_i^{(j)} = \kappa \sigma_{ij} < F\left(\left| \boldsymbol{\tau}^{(j)} \right|^2 / \tau_s{}^2 - 1 \right) > \tilde{H}(-\sigma_{jj}) + (v_{i,j} + v_{j,i}) H(\sigma_{jj}) H(\Omega^{(j)}),\ j = j_s,\ i \neq j$$

$$\gamma_i^{(j)} = 0,\ j \neq j_s,\ i \neq j,\ \left| \boldsymbol{\tau}^{(j)} \right| = \sqrt{\sum_{k \neq j} \sigma_{kj} \sigma_{kj}}$$

$$\omega^{(j)} = (\lambda v_{k,k} + 2\mu v_{j,j} - \lambda \sum_{l \neq j} \omega^{(l)}) / (\lambda + 2\mu) H(\sigma_{jj}) H(\Omega^{(j)}),\ \dot{\Omega}^{(j)} = \omega^{(j)}$$

4. Explicitly-implicit method for solving semilinear hyperbolic systems

4.1. Anisotropic models.

The obtained systems for layered and block media belong to the class of semilinear hyperbolic systems of equations. Their numerical solution can be constructed using various explicit schemes. However, in the slip mode, a nonlinear free term with a small viscosity parameter in the denominator is present. The system becomes rigid, and the usual explicit schemes will be unstable. In order to circumvent these difficulties, an explicitly implicit method is proposed. Implicit approximation applies only to those equations that contain a small parameter in the denominator of the free term, the remaining equations are approximated explicitly. The solution of the implicit difference equation is obtained analytically and is used as a corrector of the explicit "elastic" step.

Let us describe this method with the example of the equation for the slip regime for a layered or block medium in a coordinate system related to the normal to the slip-delamination plane:

$$\dot{\sigma}_{ij} \underset{i \neq j}{=} \mu(v_{i,j} + v_{j,i}) - \mu\kappa\sigma_{ij} < F\left(|\boldsymbol{\tau}|^2 / \tau_s{}^2 - 1\right) >, \; |\boldsymbol{\tau}| = \sqrt{\sum_{k \neq j} \sigma_{kj}\sigma_{kj}}$$

Implicit approximation of the first order of accuracy in time has the form:

$$(\sigma_{ij}^{n+1} - \sigma_{ij}^{n}) / \Delta t = \mu(v_{i,j}^{n+1} + v_{j,i}^{n+1}) - \mu\kappa\sigma_{ij}^{n+1} < F\left(\left|\boldsymbol{\tau}^{n+1}\right|^2 / \tau_s{}^2 - 1\right) >, \; \left|\boldsymbol{\tau}^{n+1}\right| = \sqrt{\sum_{k \neq j} \sigma_{kj}^{n+1}\sigma_{kj}^{n+1}}$$

Here, the values of the unknown quantities on the upper and lower layers of the time partition are marked with indices $n+1$ and n, the value Δt is a time step. It is assumed that the values v_i^{n+1} and σ_{jj}^{n+1} are already determined from the explicit approximation of the equations of motion and the equations for the normal components of the stress tensor without use of terms with small viscosity parameters in the denominator.

For a function $F(\Delta)$, a power-law approximation is often used:

$$F(\Delta) = \Delta^q, \; q > 0, \; \Delta = |\boldsymbol{\tau}|^2 / \tau_s^2 - 1$$

The nonlinear difference equation for the shear stress σ_{ij}^{n+1} on the upper layer in time has the form:

$$\sigma_{ij}^{n+1} = \sigma_{ije}^{n+1} - \sigma_{ij}^{n+1} < (\sigma_{ij}^{n+1} / \tau_s)^2 - 1 >^q / \zeta$$

where the small parameter ζ is $\zeta = t_0 / \Delta t \ll 1$, $t_0 = 1/(\kappa\mu)$, $\sigma_{ij(e)}^{n+1} = \sigma_{ij}^{n} + \mu(v_{i,j}^{n+1} + v_{j,i}^{n+1})\Delta t$ - “elastic” predictor. Note that we do not concretize and do not write out a scheme for calculating the "elastic" step, in particular, spatial approximations of the terms of the equation, since the choice of well-proven circuits is great [3].

For some values of q, an exact solution of the non-linear difference equation can be constructed. However, for generality, we will seek a solution of the equation in the form of an expansion in powers of a small parameter ζ, confining ourselves to one term in the expansion.

The solution for σ_{ij}^{n+1} with the chosen accuracy will look like:

$$\sigma_{ij}^{n+1} = \tau_s \left(1 + \zeta^{1/q}(\left|\sigma_{ije}^{n+1} / \tau_s\right| - 1)^{1/q} / 2\right) \operatorname{sign} \sigma_{ije}^{n+1} \quad \text{for } \left|\sigma_{ije}^{n+1} / \tau_s\right| - 1 \geq 0$$

$$\sigma_{ij}^{n+1} = \sigma_{ije}^{n+1} \quad \text{for } \left|\sigma_{ije}^{n+1} / \tau_s\right| - 1 < 0$$

The formulas obtained make sense to adjust the "elastic" stresses to the "yield point" with viscous corrections.

4.2. Isotropic models.

An analogous method can be used to obtain a whole class of correcting formulas for calculating the classical elastic-plasticity and elastic-viscoplasticity relations under various flow conditions, in particular, Wilkins' formula for calculating ideally elastic plastic flows.

The standard defining equations for stress deviators in an isotropic elastoviscoplastic medium have the following form:

$$\dot{s}_{ij} = \mu(v_{i,j} + v_{j,i}) - \mu < \sqrt{J_2} / \tau_s - 1 >^q s_{ij} / \eta$$

Here v_i are the components of the velocity vector, σ_{ij} are the components of the stress tensor, $\sigma_m = \sigma_{k,k} / 3$ is the average stress, $s_{ij} = \sigma_{ij} - \sigma_m\delta_{ij}$ are the components of the stress deviator,

$J_2 = s_{kl}s_{kl}/2$ is the second invariant of the stress deviator, τ_s is the static yield stress, η is the viscosity under dynamic loading, and q is the exponent in the expression for the stress relaxation function.

Equations of motion and the equation for the mean stress are not written out, since they do not contain small parameters in the denominator of the free term and can be approximated by standard explicit schemes.

In the numerical solution of the system of equations for deviators according to an explicit difference scheme with a step Δt under the Courant condition, the parameter $\xi = \eta/(\mu\Delta t)$ still remains small ($\xi \ll 1$) because of the smallness of the dynamic viscosity η. This small parameter is in the denominator of the viscous term, so the integration of the constitutive equations by the explicit difference scheme with the Courant time step leads to instability.

Suppose that the values of the components of the velocity vector on the $(\mathrm{n}+1)$-th time layer have already been found from the equations of motion. An implicit first order scheme for determining deviators is:

$$(s_{ij}^{n+1} - s_{ij}^{n})/\Delta t = \mu(v_{i,j}^{n+1} + v_{j,i}^{n+1}) - \mu\left(\sqrt{s_{kl}^{n+1}s_{kl}^{n+1}/2}/\tau_s - 1\right)^q s_{ij}^{n+1}/\eta$$

or

$$\xi s_{ij}^{n+1} + \left(\sqrt{s_{kl}^{n+1}s_{kl}^{n+1}/2}/\tau_s - 1\right)^q s_{ij}^{n+1} = \xi s_{ij}^{e}$$

where

$$s_{ij}^{e} = s_{ij}^{n} + \mu(v_{i,j}^{n+1} + v_{j,i}^{n+1})\Delta t$$

is an “elastic” predictor for stress deviator.

For $q = 1$, the system of algebraic equations for deviator components is solved exactly:

$$s_{ij}^{n+1} = s_{ij}^{e}\left[(1-\xi)/2 + \sqrt{(1-\xi)^2/4 + \xi\sqrt{S_1}}\right]/\sqrt{S_1}\,,\; S_1 = s_{kl}^{e}s_{kl}^{e}\big/(2\tau_s{}^2)$$

For an arbitrary value of q, the system of algebraic equations for deviators can be solved approximately for small values $\xi \ll 1$, using the expansion of the unknown values s_{ij}^{n+1} with respect to a small parameter $\xi^{1/q}$:

$$s_{ij}^{n+1} = s_{ij}^{e}\left[1 + (\sqrt{S_1} - 1)^q \xi^{1/q}\right]/\sqrt{S_1}$$

The obtained formulas for $\xi \to 0$ describe the transition from the elasto-viscoplastic to the elastoplastic model, and correspond to the well-known Wilkins correction formula of "landing" on the plasticity circle:

$$s_{ij}^{n+1} = s_{ij}^{e}\tau_s/\sqrt{s_{kl}^{e}s_{kl}^{e}/2}$$

An implicit second-order scheme with respect to time is obtained by using the mean values of the stress deviator $(s_{ij}^{n+1} + s_{ij}^{n})/2$ on the right-hand side of the implicit approximation:

$$(s_{ij}^{n+1} - s_{ij}^{n})/\Delta t = \mu(v_{i,j}^{n+1} + v_{j,i}^{n+1}) - \mu\left(\sqrt{(s_{kl}^{n+1} + s_{kl}^{n})(s_{kl}^{n+1} + s_{kl}^{n})/8}/\tau_s - 1\right)^q (s_{ij}^{n+1} + s_{ij}^{n})/(2\eta)$$

To solve this nonlinear system of equations, one can also use the method of expansion with respect to a small parameter $\xi = \eta / (\mu \Delta t)$ and obtain the result:

$$s_{ij}^{n+1} = \frac{s_{ij}^{e} - \left[S_2 / (1 + (2(S_2 - 1)\xi)^{1/q}) - 1 \right] s_{ij}^{n}}{S_2 / (1 + (2(S_2 - 1)\xi)^{1/q})}, \quad S_2 = \sqrt{(s_{kl}^{n} + s_{kl}^{e})(s_{kl}^{n} + s_{kl}^{e}) / 8} / \tau_s$$

In this formula, we can also put $\xi = 0$ and obtain an adjustment corresponding to the elastoplastic solution ("landing" on the plasticity circle):

$$s_{ij}^{n+1} = \frac{(s_{ij}^{e} + s_{ij}^{n})/2}{\sqrt{(s_{kl}^{n} + s_{kl}^{e})(s_{kl}^{n} + s_{kl}^{e})/8}} \tau_s + \left[\frac{(s_{ij}^{e} + s_{ij}^{n})/2}{\sqrt{(s_{kl}^{n} + s_{kl}^{e})(s_{kl}^{n} + s_{kl}^{e})/8}} \tau_s - s_{ij}^{n} \right]$$

Thus, if an implicit second-order accuracy scheme is used, the average deviator is first placed on the plasticity circle, and then the "landing point" is shifted by the amount of the difference between the average deviator and the value of the deviator in the old time layer.

5. Calculation examples

As an example, we consider the classical problem of passing a plane longitudinal elastic wave through a free cylindrical cavity. The amplitude of the normal stress in the wave is $\sigma_{22} = p_0$, the cavity is located in the block medium. During the passage of wave, the sliding and delamination zones are developed near the cavity (see figure 2).

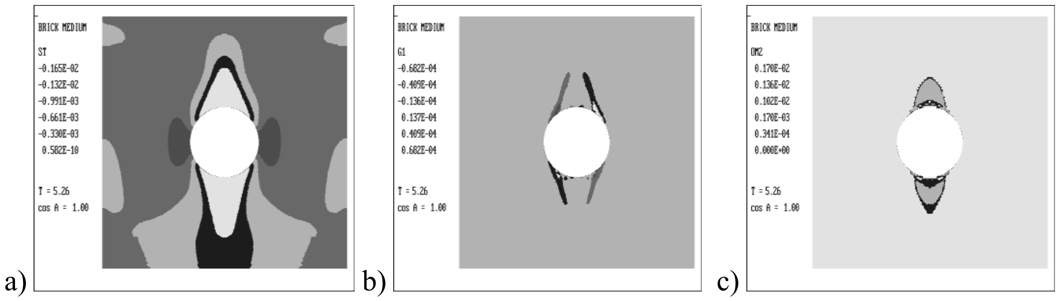
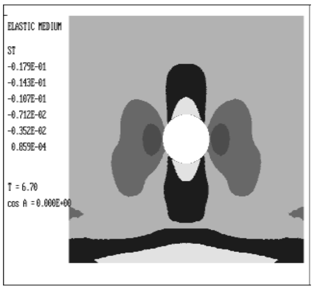
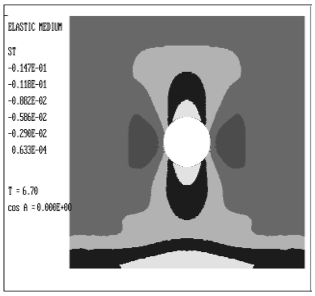


Figure 2. Block medium: a) stress level lines σ_ϑ, b) sliding zones, c) delamination zones.

In figure 2, a-c for the dimensionless parameters p_0=0.001, $\lambda = 0.33$, $\mu = 0.33$ the stress level lines, sliding zones, delamination zones are shown. It is seen that at the time after the wave passage the process of formation of the fracture zones is practically completed. The slip zone in the vicinity of the cavity and the delamination zone in the frontal and shadow parts of the cavity are appeared. We note that, for a given orientation of the layers in block medium, the nature of the relative slip and delamination in the medium under consideration is the same as in the layered medium with the horizontal orientation of the layers.

A two-dimensional problem (planar deformation) about the passage of a longitudinal wave through a cylindrical cavity in an isotropic elastic-visco-plastic or elastic-plastic ($\xi = 0$) medium was also considered. We present the results of calculations of wave propagation in the elastic-plastic medium ($\xi = 0$) for the dimensionless load $p_0 = 0.01$. The isolines of the stress component σ_ϑ are shown in figure 3, a-c at $\tau_s = 0.007$, $\tau_s = 0.005$ and $\tau_s = 0.001$. In the last case the load is 10 times more than the statical yield stress. In this case, the picture resembles the hydrodynamic flow around an obstacle (highly developed plastic flow). The stress concentration is characterized by high stress σ_ϑ values in the lateral parts of the cavity and it is decreased strongly with decreasing yield stress. We also note the known fact of displacement of the point of maximum stress concentration from the cavity boundary to the inner side zone.

a) 

b) 

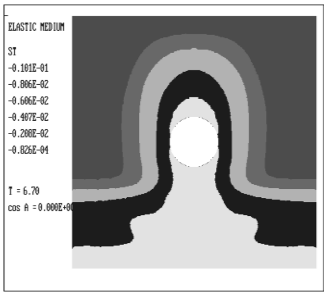
c) 

Figure 3. Elastic plastic medium: a) $\tau_s = 0.007$, b) $\tau_s = 0.005$, c) $\tau_s = 0.001$

Conclusion

A method for the numerical solution of the dynamic equations of deformable solid media with a discrete set of slip planes (in layered and block media) with nonlinear slip conditions at contact boundaries is developed. For a stable numerical solution of proposed systems of differential equations, an explicitly implicit method is developed. An explicit approximation is used for the equations of motion while an implicit approximation is applied to the constitutive equations containing a small parameter in the denominator of the free terms. Examples of numerical solutions are presented for problems on the interaction of waves with obstacles in the inelastic layered and block medium taking into account the formation of slip and delamination zones at the boundaries of structural elements.

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